Preservice Secondary Mathematics Teachers' Perceptions of Teacher Knowledge and its Sources

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Preservice secondary mathematics teachers' perceptions of teacher knowledge and of possible sources of that knowledge is investigated through examining their responses to an open-ended questionnaire. Participants place greatest emphasis on mathematics content knowledge and mathematics pedagogical knowledge and expect to gain most of their knowledge through formal preparation within the professional learning system rather than through self-study or through interactions with peers. This emphasises how important it is for schools and professional associations to provide regular formal learning opportunities, because future teachers may otherwise not be self-motivated to continue improving their skills independently of this.

PISA results from 2003 to 2018 show a consistent decline in the mathematics literacy of Australian students over the past 15 years, together with growing mathematics anxiety, particularly among girls (Thomson et al., 2019). A growing proportion of mathematics teachers are teaching out-of-field (Thomson et al, 2021), meaning students are entering university degrees with a less strong mathematical background than cohorts from previous generations. Significant attention has been given to both teacher quality standards and professional growth. What is less well known is how teachers, including preservice secondary mathematics teachers (PSMTs), perceive the notions of teacher knowledge and sources of that knowledge. We direct our focus to this area guided by the research questions:

- What are PSMTs' perceptions of teacher professional knowledge and of sources of that knowledge?
- To what extent do PSMTs recognise teacher knowledge domains and possible sources of teacher knowledge?

Considering that teachers' perceptions can influence their actions (e.g., Richardson, 1996), we believe that eliciting teachers' views is always something that we, as mathematicians and mathematics educators, should keep in mind. These elicitations can inform mathematics teacher educators in enacting a vision of teacher education and development and plausibly can improve the design of teacher education and development programmes.

Teacher Knowledge

For decades, what teachers should know and understand to teach the content effectively has been a focus of interest for researchers and teacher educators. Various teacher knowledge frameworks have been developed. For example, according to Shulman (1987), dimensions of knowledge necessary for teachers include content knowledge, general pedagogical knowledge (e.g., strategies of classroom management), curriculum knowledge (e.g., the materials and programs), pedagogical content knowledge (PCK), knowledge of learners and their characteristics, knowledge of educational contexts (e.g., the governance and financing of school districts), and knowledge of educational ends, purposes, and values. To Grossman (1990), the core of professional knowledge for teaching are general pedagogical knowledge, subject matter knowledge, pedagogical content knowledge, and knowledge of context. Ernest (1989) proposes a model where both theoretical knowledge (e.g., knowledge of mathematics and of other subject matter) and practical knowledge (e.g., knowledge of organisation and management for mathematics teaching and knowledge of the context of teaching mathematics such as knowledge of school and students taught) for teaching are identified. From Fennema and Franke's (1992) point of view, teacher knowledge includes

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knowledge of content (the concept, procedures, and problem-solving process in mathematics), knowledge of pedagogy (teaching procedures, planning, management, and motivation), knowledge of students' cognition (students' thinking, learning, and difficulties), and beliefs of teachers.

PCK—the special domain of teacher knowledge that links content, students, and pedagogy (Shulman, 1987) has received significant attention in informing teacher education and development initiatives. Based on Shulman's (1987) conceptualisation, Grossman (1990) identifies four components of teacher knowledge: teachers' "overarching conception of the purposes for teaching particular subject matter; knowledge of pupils' understanding and potential misunderstanding of a subject area; knowledge of curriculum and curricular materials; and knowledge of strategies and representations for teaching particular topics" (p. 40). Similarly, Marks (1990) proposes four components of PCK: "subject matter for instructional purposes, students' understanding of the subject matter (student learning process, typical understandings, common errors, difficulties), media for instruction in the subject matter (i.e., texts, materials), and instructional processes for the subject matter" (p. 4). An et al. (2004) define PCK as the knowledge of effective teaching comprising three components: knowledge of content, knowledge of curriculum (selecting and using appropriate textbooks and materials, understanding the goals of textbooks and curricula), and knowledge of teaching (knowing students' thinking, planning instruction, understanding the modes of presenting instruction). In this model, knowledge of teaching is the main component of PCK. Knowing students' thinking includes addressing students' misconceptions, engaging them in mathematics learning, building on their mathematical ideas, and promoting their mathematical thinking.

Another regarded teacher knowledge model is the Mathematical Knowledge for Teaching (MKT; Ball et al., 2008). MKT distinguishes between subject matter knowledge (SMK) and PCK. It consists of three domains of SMK: common content knowledge (the mathematical knowledge and skills used in settings other than teaching), specialised content knowledge (the mathematical knowledge and skills unique to teaching), and horizon content knowledge (how mathematical subjects are related in the continuum of mathematics included in the curriculum). The PCK domain includes knowledge of content and students (knowledge of how students think about, know, or learn a particular content), knowledge of content and teaching (knowledge of a mathematical content, idea or procedure and knowing pedagogical principles for teaching that content), and knowledge of content and curriculum. Within the Teacher Education and Development Study in Mathematics (TEDS-M), teacher knowledge has been differentiated according to Mathematics Content Knowledge-MCK (knowledge of subject matter), Mathematics Pedagogical Content Knowledge-MPCK (e.g., curricular knowledge, knowledge of planning for mathematics teaching and learning, analysing and diagnosing students' questions), and General Pedagogical Knowledge-GPK (Tattoo et al., 2012).

Sources of Knowledge for Teachers

There have been comprehensive approaches in teacher education and development fields to identifying sources of knowledge for teachers to improve the quality of teaching. Shulman (1987) defined sources of knowledge as "the domains of scholarship and experience from which teachers may draw their understanding" (p. 5) and identifies at least four main sources of knowledge for teachers. (1) Scholarship in subject areas: i.e., the content knowledge, understanding and skills that teachers teach. This knowledge comes mainly from the accumulated studies in the relevant subject area. (2) Educational materials and institutional contexts: i.e., the materials and structures that are created for teaching and learning including curricula, textbooks, tests and testing materials. (3) Research on schooling: i.e., the existing body of academic literature on understanding the process of teaching and learning such as empirical research findings in the fields of teaching, learning, and foundations of education. (4) The wisdom of practice: i.e., the knowledge that can be gathered from the pedagogical principles that guide and are used by exemplary teachers.

In their investigation into teachers' views about good mathematics teaching and how good teaching develops, Wilson et al. (2005) have found that the participating teachers commonly thought that good teaching "requires a sound knowledge of mathematics, promotes mathematical understanding, engages, motivates students, and requires effective management skills" and it "is developed from experience, [formal] education, personal reading and reflection, and interaction with colleagues" (p. 83). Buehl and Fives (2009) identify several major sources of knowledge for teachers based on Shulman's (1987) model:

- Formal education (college coursework, workshops, conferences, subject area classes).
- Formal bodies of knowledge: information stores (books, literature, the internet); accumulated findings (educational research).
- Observational learning (formal or informal observations of good or bad teaching).
- Collaboration or interactions: meaning construction (co-construction of knowledge through sharing and collaborating); learning from others (e.g., experts, parents, peers, and colleagues).
- Enactive experiences: personal experiences (time spent in schools as a student, the way the individual was taught); professional experiences (on-the-job, actual teaching practice, listening to students).

Prior Research

Most relevant to this investigation, the participating teachers in Mosvold and Fauskanger's (2013) study generally believed in the importance of mathematical definitions as an aspect of MKT, while the teachers did not think they needed to know definitions. The teachers in Mosvold and Fauskanger's (2014) investigation did not necessarily view the horizon content knowledge (i.e., how mathematical subjects are related in the continuum of mathematics included in the curriculum; Ball et al., 2008) as an important part of their teaching knowledge. Beginning teachers in Leong's (2014) study thought that having a sound content knowledge, classroom management skills, and motivation are the key characteristics of good teaching. Perry (2007) described a group of Australian teachers' beliefs about effective mathematics teaching and learning within a ZDM Special Issue that focused on a cross-cultural study based on interviews with participating teachers from Australia, Hong Kong SAR, Mainland China, and the USA. In the same Issue, Bryan et al. (2007) reported similarities and differences in teachers' beliefs from these four nations, while Kaiser and Vollstedt (2007) compared teachers' beliefs from the four nations reported by Bryan et al. with teachers' beliefs in Europe. According to the participating teachers from the four countries, there were commonalities in some of the attributes of effective mathematics teachers. These included competence in mathematics and necessity of in-depth understanding of the curriculum and textbooks. The latter quality was especially emphasised by teachers from Mainland China. Teachers from the US and Australia valued teachers' ability to listen to students and getting them to express themselves. Classroom management skills manifested themselves in the US teachers' responses, while this was not expressed by other teachers. All the teachers emphasised that teachers need to know their students and understand the educational needs of student. Hatisaru (submitted) investigated the extent to which a sample of in-service secondary mathematics teachers recognised the professional knowledge needed for teaching mathematics and their perceptions of its sources. Data were generated through the same questionnaire used in the current investigation. The teachers recognised the knowledge domains needed for mathematics teachers (e.g., knowledge of content, knowledge of mathematics teaching), but mostly a single knowledge dimension (e.g., knowledge of content and/or knowledge of mathematics teaching) was emphasised by the teachers as opposed to the multidimensionality of teacher knowledge. Several sources of teacher knowledge were evident in teachers' responses including both formal (workshops, conferences) and informal (peer interactions or collaboration) sources, while university coursework and educational research sources were absent.

Method

The informants for this study were PSMTs who expressed interest in participating in a research project aiming to enhance their representational competence. The research was advertised in two mathematics education units, in the Bachelor of Education (BEd; 19 enrolled) and Master of Teaching (MTeach; 11 enrolled) programmes, at the authors' university. Within this research, interested PSMTs completed an open-ended questionnaire containing eight items sourced from the literature. Relevant to this paper are Items 1 to 4 (adapted from An et al., 2004) where the aim is to access the PSMTs' perceptions of the types of professional knowledge that teachers of mathematics should have (Item 1) and how that knowledge is developed (Item 3), with a specific focus on how knowledge of students is gained (Item 4):

Item #1: What type of professional knowledge should a teacher of mathematics have?

Item #2: How important is it for teachers to have this knowledge?

Item #3: How do teachers continue to enhance their professional knowledge?

Item #4: How do teachers know about their students' strategies and understanding of a particular mathematical content?

From the entire population enrolled in two units, six second-year BEd (four male and two female) and six first-year MTeach (three male and three female) PSMTs (n = 12) voluntarily completed the questionnaire. Nine of them were majoring in mathematics while the other three were studying a minor in mathematics. Participants are coded as PSMT 1, PSMT 2, PSMT 3, etc. to protect their identity.

The data analysis was conducted by the first author and coding was reviewed by the second author. The examination of the words and language used by the respondents was the key aspect of data analysis. We therefore content analysed the PSMTs' responses to four questionnaire items to discover and describe their perceptions, informed by the categories of teacher knowledge, and its sources, identified in the relevant research literature presented in the second section. We were also *open* to adding more knowledge dimensions if they should arise within the data. Some responses were coded in more than one category to not lose the richness of the data. Once the coding was completed, the frequency of teacher knowledge dimensions and sources of knowledge manifested in the PSMTs' responses were counted and presented in Tables 1 and 2. We present the findings below under two sections. Using quotes from the PSMTs' responses, we aim to capture the perceptions of the PSMTs and to describe the trends in their descriptions. Within these quotes, knowledge dimension coding is given in brackets, and italics are added by the authors.

Findings

Perceptions of Teacher Knowledge

Table 1 captures the teacher knowledge dimensions which manifested themselves in the PSMTs' responses to relevant questionnaire items. The PSMTs put greater value on teacher professional knowledge (Item #1) and mostly indicated the importance of understanding mathematical content (17 occurrences), knowledge of mathematics pedagogy (9 occurrences), and knowledge of general pedagogy (6 occurrences) (Items #1 and #2). Typical examples included, in responding to Item #1:

> A maths teacher should firstly have *a basic understanding of and knowledge of mathematics* [K of content], and in my opinion, should have done maths ATAR in high school as a minimum. In addition, I think it's important that a maths teacher *has some knowledge of instructional practice when it comes to teaching maths* to know how students may learn best [K of maths pedagogy]. (PSMT 10)

> Probably a decent grasp on *what is being taught* [K of content]. As well as an understanding of *the Australian/WA curriculum* [K of curriculum]—*OLNA, NAPLAN, etc*. [K of context] (PSMT 7)

Table 1

Dimensions of Teacher Knowledge Revealed in PSMTs' Responses

Item #1	Item #2
Knowledge of content (11)	Knowledge of content (6)
Knowledge of mathematics pedagogy (5)	Knowledge of mathematics pedagogy (4)
Knowledge of curriculum (3)	Knowledge of pedagogy (4)
Knowledge of context (2)	Knowledge of students (1)
Knowledge of pedagogy (2)	
Knowledge of students (1)	

Note. Numbers in brackets indicate the number of occurrences.

Additionally, three participants foregrounded *content-free* mathematical competencies and applications of particular mathematics concepts, or real-life applications of mathematics:

> How to *use mathematics to solve* simple but useful *problem*s, a little bit of the philosophy of mathematics (e.g., what is it? what sort of things do mathematicians fight over? what's the difference between a conjecture and a theorem?) and *the ability to do basic mathematical proofs* … [K of content] (PSMT 8)

> A maths teacher must fully *understand a concept*, *different ways this concept can be taught* as students don't all learn the same way and must ideally include *relevant applications of this concept* [K of content; K of maths pedagogy] (PSMT 6)

In responding to Item #2, where the PSMTs were prompted about the importance for teachers to have professional knowledge needed to teach mathematics, seven of them reinforced what teachers would know or understand, as in these example responses:

> A stronger *knowledge of the content* [K of content] as well as *different styles about how to convey that information* [K of maths pedagogy] means they can assist a larger number of students. (PSMT 4)

> It's important that *teachers understand the concepts of maths* [K of content] and *different ways of teaching maths* to be able to effectively convey that knowledge to students [K of maths pedagogy]. (PSMT 10)

Out of twelve, five PSMTs underlined the importance of teachers having knowledge of pedagogy. The ways they described this kind of knowledge can be understood as knowing how to engage students in mathematics learning and knowledge of different teaching strategies, as described (for example) by Fennema and Franke (1992). Whilst three of the PSMTs' responses referred to the knowledge of how to motivate students towards learning (PSMT 2, Item #1; PSMT 5, Item #2; and PSMT 12, Item #2), the other responses addressed to knowing "how to teach" (PSMT 2, Item #2), "as well as [knowing] different teaching styles" (PSMT 4, Item #1).

> I believe this grounding [teacher professional learning] is essential, as it provides the teacher with the foundational understandings that students require. Learning *how to teach maths*, *using manipulatives*, *different representations*, *games* [K of maths pedagogy; K of content] and *stimulating student interest* [K of pedagogy] will enable maths teachers to be more effective. (PSMT 12, Item #2)

Perceptions of Sources of Teacher Knowledge

PSMTs' suggestions of possible knowledge sources for teachers are presented in Table 2. In responding to Item #3, almost all PSMTs consider formal preparation as one of the major sources of teacher knowledge. This includes professional development opportunities such as seminars, workshops, professional learning days, online MOOCs, as well as postgraduate studies and university graduate certificates. Some representative examples are:

> By doing *professional learning* (may it be on PD days [Formal preparation], *seeking assistance from colleagues* [Interactions or collaboration] or *prior reading regarding a topic* [Formal bodies of knowledge]) (PSMT 6)

> Teachers enhance their knowledge by completing *online professional learning courses*, completing professional learning *workshops*, *completing graduate certificates in particular subjects*, [Formal preparation], *joining professional associations* [Formal bodies of knowledge], such as … (PSMT 12)

Table 2

Sources of Teacher Knowledge Revealed in PSMTs' Responses

Item #3	Item #4
Formal preparation (18)	Professional experiences (18)
• professional learning days, seminars, courses, activities (12); postgraduate studies (5); external studies (1)	• conducting (formative) assessment (7); conversations with students (5); observations (4); on-the-job (2)
Formal bodies of information (5)	Interactions or collaboration (3)
• books (2); research (1); curriculum (1); professional associations (1)	• communication for growth and understanding others (1); cooperation with other teachers (1); interacting in the school (1)
Self-study/reflection (5)	Formal preparation (2)
• self-study (4); self-reflection (1)	• studying at university (1); learnings at university (1)
Interactions or collaboration (3)	Formal bodies of information (2)
• from others (1); from peers (1); communicating with colleagues (1)	• checklists (1); handbooks showing students' misconceptions (1)
Professional experiences (1)	Self-study/reflection (2)
• interacting with students	• educate themselves (1); be receptive to feedback (1)

Note. Numbers in brackets indicate the number of occurrences.

Whilst the PSMTs predominantly regarded formal preparation as a way of gaining professional knowledge in general (Item #3), they perceived professional experiences as one of the main sources of ways for acquiring knowledge of students' strategies and understanding regarding a particular mathematical content (Item #4), like participants in Buehl and Fives (2009). Many responses within this group were grouped into conducting (mostly formative) assessment, listening to, or observing students. Perhaps these are best reflected in the following words:

> *Talk to them*, *look at their working*, *look over their shoulder* at the work. (PSMT 8)
>
> Diagnostic, formative and summative assessment. Asking students not only to answer questions but to give justification for their answers. (PSMT 9)
>
> Continuously checking student understanding in the classroom through *formative assessments* is important to be able to gauge student understanding of a particular topic. (PSMT 10)

Fewer PSMTs perceived formal preparation as a source of acquiring knowledge of student thinking and understanding. PSMT 7 wrote:

> Perhaps from *their own experiences* [on-the-job] as well as those they have taught, and the information they have learned from university [Formal preparation].

Discussion and Conclusions

PSMTs place greatest emphasis on mathematics content knowledge and mathematics pedagogical knowledge, two components of teacher knowledge that have been generally considered as key factors for effective mathematics teaching and students' mathematics learning (e.g., An et., 2004). This is perhaps to be expected in a subject such as mathematics, which faces unique challenges among school disciplines in terms of student engagement, student anxiety, the wide range of applications, and cultural expectations. It is therefore expected that PSMTs would highly value content knowledge as well as specialised knowledge on how to engage students with mathematics. However, we do not know preservice teachers' values of different knowledge domains across different subject disciplines and whether mathematics is an outlier. The PSMTs expect to gain most of their knowledge through formal preparation within the professional learning system rather than through self-study or through interactions with peers. This emphasises how important it is for schools and professional associations to provide regular formal learning opportunities, because teachers may otherwise not (usually) be self-motivated to continue improving their skills independently of this. Saying that, we would like to see a study done of the professional development opportunities in Australia for mathematics teachers, to determine whether the range of topics being taught matches with what preservice, and practising, mathematics teachers feel are the most important areas to know. What gaps, if any, are there in professional development opportunities, and are all the knowledge domains being adequately explored by the professional service providers?

Our research has looked into the expectations of knowledge of PSMTs, before they have yet had to put their knowledge into practice. PSMTs put a large emphasis on content knowledge in their training, however, as noted by Buehl and Fives (2009), the content knowledge of mathematics tends to remain stable and consistent over time, in comparison to other subjects such as social studies which may change more rapidly. We would expect, then, that as participants progress from preservice to in-service, they begin to rate pedagogical knowledge and context more highly than content knowledge as observed in Hatisaru (submitted). Future work could follow these students as they progress to becoming in-service teachers and look at how their answers to our questionnaire items, or to those similar ones, would change with their professional practice.

The results from only twelve PSMTs, in one state of Australia within one single university may not apply to different contexts. The educational background of the participants was not taken into account—both their prior level of knowledge starting university and the country where they were educated. In a larger study we may also have considered whether gender influences the answers of the PSMTs, together with their mathematics training background and differences between BEd and MTeach students. Furthermore, findings are limited to PMSTs' responses to four questionnaire items; we do not have, for example, interview data which may expand on their written responses.

On a final note, teacher knowledge is a multi-dimensional construct, including several knowledge components, and all these components overlap. For example, knowledge about the applications of mathematics spans the categories of content knowledge, pedagogical content knowledge, and knowledge of context. Furthermore, different mathematics educators or researchers define these components differently. The knowledge of context category has typically been considered as educational context by previous authors but could be expanded to include mathematical context, as in Ernest (1989), to encapsulate those knowledge domains regarding the purpose and philosophy of mathematics. Researchers aiming to understand teachers' beliefs about teacher knowledge need to be prepared for the complexity of the construct of teacher knowledge.

References

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, *7*, 145–172.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal for Teacher Education*, *59*(5), 389–407. https://doi.org/10.1177/0022487108324554

Buehl, M. M., & Fives, H. (2009). Exploring teachers' beliefs about teaching knowledge: Where does it come from? Does it change? *Journal of Experimental Education*, *77*, 367–408.

Bryan, C. A., Wang, T., Perry, B., Wong, N-Y., & Cai, J. (2007). Comparison and contrast: Similarities and differences of teachers' views of effective mathematics teaching and learning from four regions. *ZDM Mathematics Education*, *39*(4), 329–340. https://doi.org/10.1007/s11858-007-0035-2

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching*, *15*(1), 13–33.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*, (pp. 147–164). Macmillan Publishing Company.

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education*. Teachers College Press.

Hatisaru, V. (submitted). Secondary teachers' perceptions of teacher knowledge and its sources. *Quadrante*.

Kaiser, G., & Vollstedt, M. (2007). Teachers' views on effective mathematics teaching: Commentaries from a European perspective. *ZDM Mathematics Education*, *39*, 341–348.

Leong, K. E. (2014). What are the important attributes of good mathematics teaching? *Asia-Pacific Educational Research*, *24*(1), 211–223. https://doi.org/10.1007/s40299-014-0173-6

Marks, R. (1990). Pedagogical content knowledge: From a mathematical case to a modified conception. *Journal of Teacher Education*, *41*(3), 3–11.

Mosvold, R., & Fauskanger, J. (2013). Teachers' beliefs about mathematical knowledge for teaching definitions. *International Electronic Journal of Mathematics Education*, *8*(2–3), 43–61.

Mosvold, R., & Fauskanger, J. (2014). Teachers' beliefs about mathematical horizon content knowledge. *International Journal for mathematics teaching and learning*, 1–16.

Perry, B. (2007). Australian teachers' views of effective mathematics teaching and learning. *ZDM Mathematics Education*, *39*(4), 271–286. https://doi.org/10.1007/s11858-007-0032-5

Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In J. Sikula (Ed.), *Handbook of research on teacher education* (pp. 102–119). Macmillian.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1–22.

Tatto M. T., Schwille J., Senk S. L., Ingvarson L., Rowley G., Peck R., . . . Reckase M. (2012). *Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA teacher education and development study in mathematics (TEDS-M)*. IEA.

Thomson, S., De Bortoli, L., Underwood, C., & Schmid, M. (2019). *PISA 2018: Reporting Australia's results. Volume I: Student performance*. Melbourne: Australian Council for Educational Research (ACER). https://research.acer.edu.au/ozpisa/35

Thomson, S., Wernert, N., Buckley, S., Rodrigues, S., O'Grady, E., & Schmid, M. (2021). *TIMSS 2019 Australia. Volume II: School and classroom contexts for learning*. Melbourne: Australian Council for Educational Research (ACER). https://doi.org/10.37517/978-1-74286-615-4

Wilson, P. S., Cooney, T. J., & Stinson, D. W. (2005). What constitutes good mathematics teaching and how it develops: Nine school teachers' perspectives. *Journal of Mathematics Teacher Education, 8*, 83–111.